MathString Zentangle® Workshop:

The Golden Ratio and the Elegance of Limits

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About the class designers:

Pilar Pulido, CZT
Completely hooked by Zentangle when Pilar discovered it, she didn’t doubt at any second to become a CZT (April 2017, seminar #26). Her professional career is linked to the Information Technologies where she is still working at the same time as she is starting up her dream of a Zentangle business. She had been always worried about her creativity but through Zentangle has found her skills. She is also a LEGO Serious Play facilitator, another method full of innovation.
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Lynn Mead, CZT
No matter where her life has lead, Lynn has approached it as a Creative Maker. Whether working at a job, or embracing her role as wife and mother, creativity has been an essential element. Her work experiences have ranged from commercial printing to software testing but most of her career she owned her own freelance design company. Her artistic endeavors have ranged from fused glass to block printing, fiber arts to metal embossing, in other words, her interests are many. In 2013 Lynn was introduced to Zentangle and it was like finding the missing link that connected all her creative efforts. She became a Certified Zentangle Teacher in 2014 (Class 14). Since then, Zentangle has become the central focus of all her creative endeavors. She is the designer of the Tangle Deck line of tangling resources and has taught for several years at TangleU, a continuing education conference for CZTs.
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MathString Zentangle® Workshop: the Golden Ratio and the Elegance of Limits

What is it about some objects that make them aesthetically pleasing? It is thought one answer to this question is the Golden Ratio. The Golden Ratio is a mathematical relationship that exists in art, shapes, nature, and patterns.

The math behind the Golden Ratio is based on the Fibonacci Sequence. The Fibonacci Sequence is simply this; each number in the sequence is the sum of the previous two. (Do I see a pattern here?)

\[ 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots \]

\[ 1 + 1 = 2, \ 2 + 1 = 3, \ 3 + 2 = 5, \ 5 + 3 = 8, \ldots \]

The Golden Ratio is defined as follows:

\[
\frac{a}{b} = \frac{b}{c}
\]

where \( a + b = c \)

the ratio of the length of segment \( a \) to \( b \)

is the same as

the ratio of the length of segment \( b \) to \( c \)

Note you can substitute any 3 consecutive numbers from the Fibonacci sequence for \( a \), \( b \), and \( c \).

The Golden Ratio is also seen in the Golden Rectangle. The ratio of the shorter side to the longer side is 1 to 1.618. It’s interesting to note that if you define the largest square possible inside a golden rectangle, what is left over is a smaller golden rectangle. This process can be repeated with each golden rectangle, and each square maintains the golden ratio with the previous square.

The Golden Spiral is based on the golden ratio of the squares within the golden rectangle. Adding a quarter arc to each square results in a Golden Spiral.
Fibonacci numbers and the Golden Ratio can be seen all around us from the microscopic to the macroscopic. Here are a few examples:

**DNA**

Each twist if a DNA molecule fits in a rectangle measuring in the Fibonacci ratio of 34:21 angstroms.

**The way seeds are packed into seed heads**

The head of a sunflower has two series of opposing spirals. The number of spirals is in general either 21 and 34, or 34 and 55, or 55 and 89 etc. The same is true for pine cones, 5 and 8 etc.

**The proportions of the bones in your fingers**

The length of the first and second bones added together equals the length of the third. This allows you to fold your fingers into a compact spiral to form a fist.

**In Music**

Fibonacci numbers appear in the modern scale (2, 3, 5, 8, 13)

groups of 2 and 3 black keys = 5 black keys
8 white keys
total of 13 keys

**The shape of hurricanes and spiral galaxies**

If you’d like to learn more about the Fibonacci Numbers and the Golden Ratio see the Resources section.

The Golden Ratio along with other maths seem to be important in defining the framework of our universe or to put it in Zentangle terms “the strings” that determine how some things look and act. We are not always aware of their existence because, like the strings on a Zentangle tile they disappear beneath the surface, but they are there and they beautifully demonstrate the Zentangle concept of the “elegance of limits”.

It is the Golden Spiral we have chosen as our “math string” for this class project, but don’t worry, we’ve done the math for you!
Materials:
- Precut Fabriano Pergamon Paper, weight: 230g/m², color: ivory (guess what, it’s a golden rectangle)
- Two pre-printed templates (one right facing, one left facing)
- Pencil
- Various colors of Micron (or other) Pens (01, PN)

Description:
The exercise for this class is done on a sheet of paper cut to a golden rectangle. We have chosen 9 cm x 14.5 cm (3.5 in x 5.625 in) as it is based on the size of a standard Zentangle tile with additional length added to make it the correct ratio for a Golden Rectangle. The paper we are using is Fabriano Pergamon which is a translucent parchment so allows for tracing from a template placed beneath. Think of it as a pre-strung tile. Provided are spiral templates for both directions at the correct size for the provided paper.

NOTE: A word about the paper.
The paper we are using is Fabriano Pergamon, weight: 230g/m², color: ivory. It is a parchment that has some texture and a special combination of translucence and opacity. Both sides are the same so you can draw on either side (more about that later.) The good news is that it is available in both Europe and Canada. The bad news is that we have not found a source in the United States. As of now we are still looking for either a source for this paper or an adequate substitute. In a pinch, a sheet of light weight drawing paper might work, you’ll have to experiment.

As demonstrated by the templates each rectangle consists of a series of ever smaller squares based on the Golden Ratio. This series of squares that make up the golden rectangle is our reticulum. Each of the individual squares is considered a fragment. These fragments are rotated to the right as you travel clockwise around the rectangle or left if going anti-clockwise. The rotation of the arc in each fragment is what creates the spiral. Additional arcs based on the golden ratio have been added to create a double spiral. Each fragment will be filled with lines and shapes (tangles).

Example 1: (left) a representation of the template and (right) the basic fragment

Example 2 - (left) Golden Spiral created using Phicops, and (right) broken down into fragments, each rotated 90°
How to draw a Phicops spiral

Begin by taping your paper over the template. The paper is translucent so gives the effect of a pre-strung tile. Taping will also keep the paper from shifting as you rotate the tile.

The lines on the templates are for reference. You can use only the ones that seem relevant to you and ignore the others.

To draw Phicops use both the inner and outer spiral lines as guides.

Start at the outer spiral and draw the initial curved lines extending to the inner spiral. Note the lines are farther apart on the outer spiral than they are on the inner spiral.

Don’t forget to rotate your paper as you draw.

Next, draw a line from the top of each curve back and down to the previous line creating a sawtooth effect.

To complete the basic Phicops, draw a line from the points along the outer spiral forward, connecting to the next shape.

Finally, connect the lines along the inner spiral with a slight curve.

Phicops is ready for embellishing and tangles.

Note: Laura Harms initial blog post on Phicops can be accessed via tanglepatterns.com (and you'll find there is a connection to the golden ratio!)
A mosaic of ideas using the Golden Spiral
Drawings by Lynn Mead and Pilar Pulido

But wait, there’s more! The best part is still to come.
Continue to the next page to discover the surprise...
Now for the surprise...

One of the reasons the Fabriano Pergamon paper was chosen for this project is its combination of opacity and translucence. In addition to the ability to see the template and use that as a string, one is also able to add color and pattern on the back. It will be barely noticeable from the front in normal light but magic happens when you view the drawing lighted from the back.

Keep this in mind when embellishing your spiral. Add color and tangles to both the front and back for the full effect.

Here are a few examples.

This version of Phicops was embellished with Onamato, color and graphite shading.
In addition, white gel pen was used to add Sandswirl to the background.

Then color was added to the back.
In normal light when viewed from the front the color is barely noticeable but helps create the subtle design of the white gel pen (see above)

This is the view when the drawing is lit from behind.
Notice that since the white gel pen is opaque it blocks the light and shows up as a darker gray color.

A note about adding color:

We tested adding color with a variety of markers. Copic, Fabrico, Tombow, and Identipen all worked on this paper. I do not recommend using a water brush to blend water based markers, however, the Tombow blending pen worked well. We also found that the texture of the paper works very well with pastel pencils. We did not test colored pencils.

Gel pens work for adding colored line work but because they are opaque they will be a solid dark line when back lit. This can be an interesting effect or a disappointment if you don’t take that into account.
Here are a few more examples with back lighting.

**Natural Light**  
**Back lit**

Drawing by Lynn Mead  
Added color was Identipen

Drawing by Pilar Pulido  
Added color was done with pastel pencil.

Drawing done by Joanna Quincey  
European CZT Conference 2018,  
Cork, Ireland

Drawing done by Marguerite Samama  
European CZT Conference 2018,  
Cork, Ireland  
Note the Phicops variation

The possibilities are endless and you are guaranteed to create a drawing of divine proportions!
Resources

Books

The Golden Section, Nature’s Greatest Secret
by Scott Olsen

Sacred Geometry,
by Miranda Lundy

Phicops story and step-out

A link to the story of how Phicops was named and how to draw it.

Web resources

https://www.mathsisfun.com/numbers/nature-golden-ratio-fibonacci.html

This site has a fun interactive demonstration of how the Fibonacci sequence and the Golden Ratio help plants to pack the maximum number of seeds into their seed heads.


This site has examples of the Fibonacci sequence and the Golden Ratio in nature.

https://www.youtube.com/watch?v=ahXIMUkSXX0
https://www.youtube.com/watch?v=IOIP_Z_-0Hs

Fun YouTube videos about the Fibonacci numbers and plants.

We have provided this handout for free but if you’d like to help us defray the costs of developing the class and providing the download, you can send a donation to lynn@atanglersmind.com through PayPal. Here’s a link that explains how to do that:
